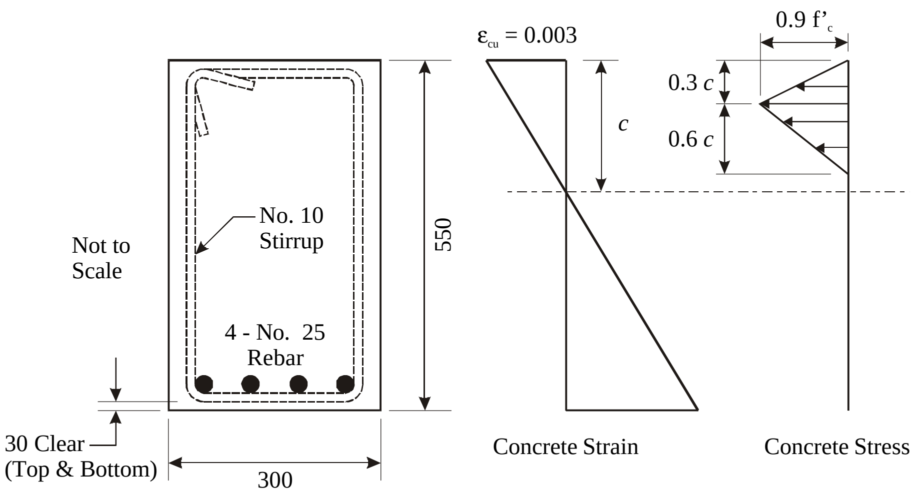
Question 1:



Given: $f'_c := 30 \cdot MPa$ $\quad f_y := 400 \cdot MPa$ $\quad E_s := 200000 \cdot MPa$ $\quad \varepsilon_{cu} := 0.003$

- Beam dimensions: $h := 550 \cdot mm$ $\quad b := 300 \cdot mm$ $\quad cc := 30 \cdot mm$ $\quad d_{st} := 10 \cdot mm$

$n_{bar} := 4$ $\quad d_b := 25 \cdot mm$ $\quad A_{s\_bar}(d_b) = 500\,mm^2$ $\quad A_s := n_{bar} \cdot A_{s\_bar}(d_b)$ $\quad A_s = 2000\,mm^2$

Solution:

- Effective depth "d": $d := h - (cc + d_{st}) - 0.5 \cdot d_b$ $\quad d = 497.5\,mm$

- Location of n.a. $C_c = T$ $\quad$ Assume steel yields $\quad f_s = f_y$

$$C_c = 0.5 \cdot (0.9 \cdot f'_c) \cdot [(0.3 \cdot c + 0.6 \cdot c) \cdot b] \qquad T = A_s \cdot f_y$$

$$0.5 \cdot (0.9 \cdot f'_c) \cdot [(0.3 \cdot c + 0.6 \cdot c) \cdot b] = A_s \cdot f_y$$

$$c := \frac{A_s \cdot f_y}{0.405 \cdot f'_c \cdot b} \qquad c = 219.5\,mm$$

- Check yielding: $\varepsilon_s := \varepsilon_{cu} \cdot \left(\frac{d - c}{c}\right)$ $\quad \varepsilon_s = 0.0038$

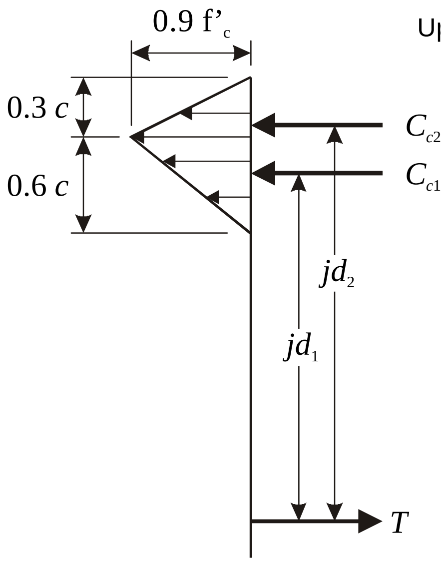
$$\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 0.002 \qquad \frac{\varepsilon_s}{\varepsilon_y} = 1.9001$$

- OK, steel yields

- Nominal moment resistance:

Lower triangle: $C_{c1} := 0.5(0.9 \cdot f'_c) \cdot [(0.6 \cdot c) \cdot b]$ $\quad C_{c1} = 533.33\,kN$

$$jd_1 := d - \left(0.3 \cdot c + \frac{1}{3} \cdot 0.6 \cdot c\right) \qquad jd_1 = 387.8\,mm$$



Upper triangle: $C_{c2} := 0.5\left(0.9 \cdot f'_c\right) \cdot [(0.3 \cdot c) \cdot b]$ $C_{c2} = 266.67\,kN$

$jd_2 := d - \left(\frac{2}{3} \cdot 0.3 \cdot c\right)$ $jd_2 = 453.6\,mm$

Check:

$T := A_s \cdot f_y$ $T = 800\,kN$

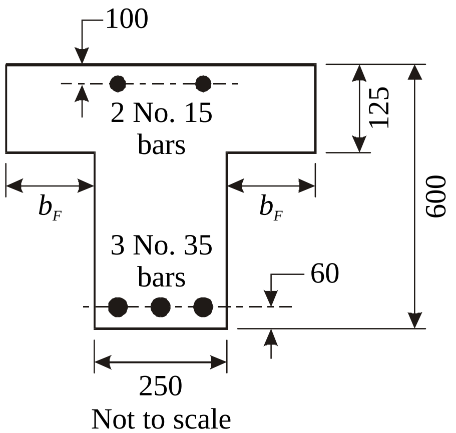
$T - \left(C_{c1} + C_{c2}\right) = 0\,kN$ OK

$\Sigma M_T = 0$ $M_{n1} := C_{c1} \cdot jd_1$ $M_{n1} = 206.81\,kN \cdot m$

$M_{n2} := C_{c2} \cdot jd_2$ $M_{n2} = 120.96\,kN \cdot m$

$M_n := M_{n1} + M_{n2}$ $M_n = 327.8\,kN \cdot m$

Question 2:



$\phi_c := 0.65 \qquad \phi_s := 0.85$

Given: $f'_c := 25 \cdot MPa \qquad f_y := 500 \cdot MPa \qquad E_s := 200000 \cdot MPa \qquad \varepsilon_{cu} := 0.0035$

- Beam dimensions: $h := 600 \cdot mm \qquad b_w := 250 \cdot mm \qquad h_F := 125 \cdot mm$

$cc := 60 \cdot mm \qquad cc' := 100 \cdot mm$

$n_{bar} := 3 \qquad d_b := 35 \cdot mm \qquad A_{s\_bar}(d_b) = 1000\,mm^2 \qquad A_s := n_{bar} \cdot A_{s\_bar}(d_b) \qquad A_s = 3000\,mm^2$

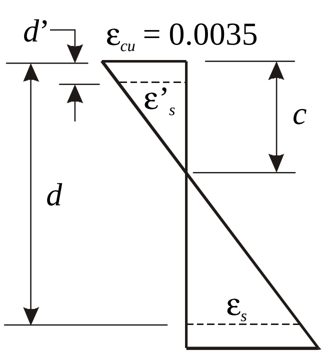
$n_{bar'} := 2 \qquad d'_b := 15 \cdot mm \qquad A_{s\_bar}(d'_b) = 200\,mm^2 \qquad A'_s := n_{bar'} \cdot A_{s\_bar}(d'_b) \qquad A'_s = 400\,mm^2$

Solution:

$\alpha_1 := 0.85 - 0.0015 \cdot \frac{f'_c}{MPa} \qquad \alpha_1 = 0.8125 \qquad \beta_1 := 0.97 - 0.0025 \cdot \frac{f'_c}{MPa} \qquad \beta_1 = 0.9075$

- Effective depths: $d := h - cc \qquad d = 540\,mm \qquad d' := cc' \qquad d' = 100\,mm$

- Location of n.a. $\varepsilon_y := \frac{f_y}{E_s} \qquad \varepsilon_y = 0.0025$



$\varepsilon_s := 2.5 \cdot \varepsilon_y \qquad \varepsilon_s = 0.0063$

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_s}{d - c}$$

$c := \varepsilon_{cu} \cdot \frac{d}{(\varepsilon_{cu} + \varepsilon_s)} \qquad c = 193.85\,mm$

$a := \beta_1 \cdot c \qquad a = 175.92\,mm$

- Strain in compressive reinforcement

$$\varepsilon'_s := \varepsilon_{cu}\cdot\left(\frac{c-d'}{c}\right) \qquad \varepsilon'_s = 0.00169 \qquad f'_s := \varepsilon'_s\cdot E_s \qquad f'_s = 338.89\,\text{MPa}$$

Required flange width:

- Area of steel to balance compression steel:

$$\phi_s\cdot A_{s1}\cdot f_y = A'_s\cdot\left(\phi_s\cdot f'_s - \phi_c\cdot\alpha_1\cdot f'_c\right)$$

$$A_{s1} := A'_s\cdot\frac{\left(\phi_s\cdot f'_s - \phi_c\cdot\alpha_1\cdot f'_c\right)}{\left(\phi_s\cdot f_y\right)} \qquad A_{s1} = 258.7\,\text{mm}^2$$

- Area of steel to balance concrete compression in web:

$$\phi_s\cdot A_{s2}\cdot f_y = \left(\phi_c\cdot\alpha_1\cdot f'_c\right)\cdot\left(a\cdot b_w\right)$$

$$A_{s2} := \phi_c\cdot\alpha_1\cdot f'_c\cdot a\cdot\frac{b_w}{\left(\phi_s\cdot f_y\right)} \qquad A_{s2} = 1366.3\,\text{mm}^2$$

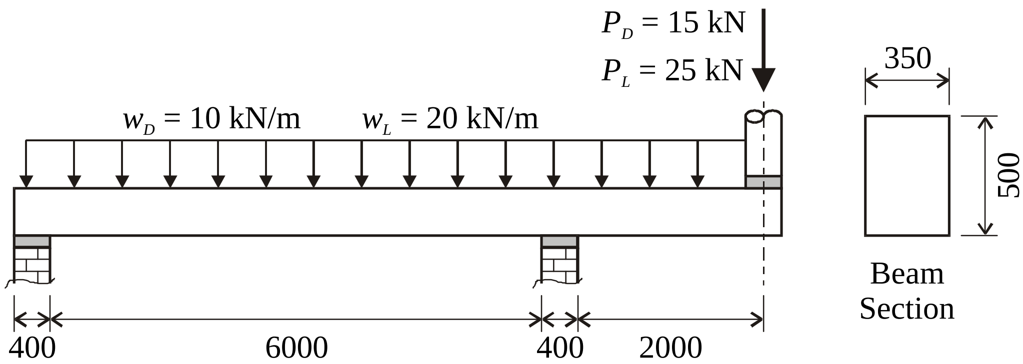
- Remaining steel area to be balanced by concrete compression in flanges:

$$A_{s3} := A_s - A_{s1} - A_{s2} \qquad A_{s3} = 1375.06\,\text{mm}^2$$

$$\phi_s\cdot A_{s3}\cdot f_y = \left(\phi_c\cdot\alpha_1\cdot f'_c\right)\cdot\left[h_F\cdot\left(2\cdot b_F\right)\right]$$

$$b_F := \frac{1}{2}\cdot\phi_s\cdot A_{s3}\cdot\frac{f_y}{\left(\phi_c\cdot\alpha_1\cdot f'_c\cdot h_F\right)} \qquad b_F = 177\,\text{mm}$$

Question 3: $\phi_c := 0.65$ $\phi_s := 0.85$ $\alpha_D := 1.25$ $\alpha_L := 1.5$ $\gamma_c := 2400 \cdot \frac{kg}{m^3}$



Given: $f'_c := 30 \cdot MPa$ $f_y := 400 \cdot MPa$ $E_s := 200000 \cdot MPa$ $\varepsilon_{cu} := 0.0035$

- Beam dimensions: $h := 500 \cdot mm$ $b := 350 \cdot mm$ $cc := 30 \cdot mm$ $d_{st} := 10 \cdot mm$

  $b_{sup} := 400 \cdot mm$ $l_{n1} := 6000 \cdot mm$ $l_{n2} := 2000 \cdot mm$

- Superimposed loading: $w_D := 10 \cdot \frac{kN}{m}$ $w_L := 20 \cdot \frac{kN}{m}$ $P_D := 15 \cdot kN$ $P_L := 25 \cdot kN$

Solution:

$\alpha_1 := 0.85 - 0.0015 \cdot \frac{f'_c}{MPa}$ $\alpha_1 = 0.805$ $\beta_1 := 0.97 - 0.0025 \cdot \frac{f'_c}{MPa}$ $\beta_1 = 0.895$

- Balanced Reinforcement Ratio: $\rho_b := \frac{\phi_c \cdot \alpha_1 \cdot f'_c \cdot \beta_1}{\phi_s \cdot f_y} \cdot \left(\frac{700}{700 + \frac{f_y}{MPa}} \right)$ $\rho_b = 0.0263$

- Effective depth: Assume $d_b := 30 \cdot mm$ $d := h - (cc + d_{st}) - 0.5 \cdot d_b$ $d = 445\,mm$

Factored loading:

Self weight: $w_{Dsw} := (b \cdot h) \cdot (\gamma_c \cdot g)$ $w_{Dsw} = 4.12 \frac{kN}{m}$

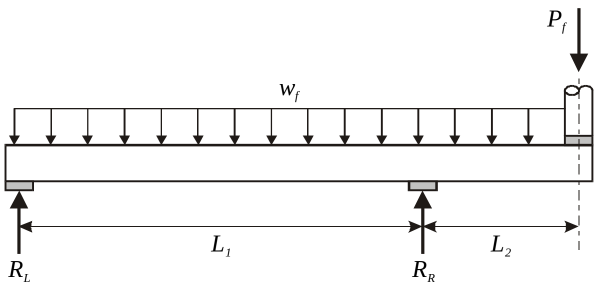
$w_f := \alpha_D \cdot (w_{Dsw} + w_D) + \alpha_L \cdot w_L$ $w_f = 47.65 \frac{kN}{m}$

$P_f := \alpha_D \cdot P_D + \alpha_L \cdot P_L$ $P_f = 56.25\,kN$

Simple span lengths:

$L_1 := l_{n1} + 2(0.5 \cdot b_{sup})$ $L_1 = 6.4\,m$ $L_2 := l_{n2} + 0.5 \cdot b_{sup}$ $L_2 = 2.2\,m$

FBD:



$\Sigma M_L = 0$

$$w_f \cdot \frac{(L_1 + L_2)^2}{2} + P_f \cdot (L_1 + L_2) - R_R \cdot L_1 = 0$$

$$R_R := \frac{w_f \cdot \frac{(L_1 + L_2)^2}{2} + P_f \cdot (L_1 + L_2)}{L_1}$$

$R_R = 350.9\,\text{kN}$

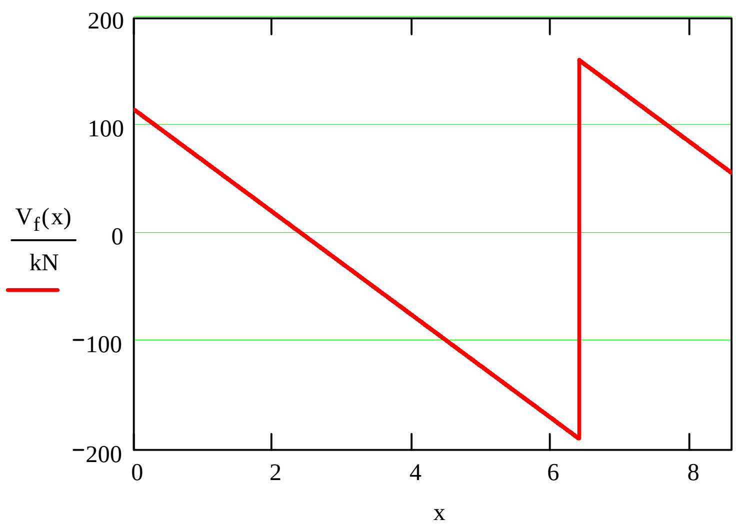
$\Sigma F_v = 0 \qquad R_L - w_f \cdot (L_1 + L_2) + R_R - P_f = 0$

$R_L := w_f \cdot L_1 + w_f \cdot L_2 - R_R + P_f \qquad R_L = 115.12\,\text{kN}$

Shear Diagram:

$$V_f(x) := \begin{cases} R_L - w_f \cdot x & \text{if } x \le L_1 \\ R_L - w_f \cdot x + R_R & \text{otherwise} \end{cases}$$

$x := 0.0 \cdot \text{mm}, 1 \cdot \text{mm} .. L_1 + L_2$



$V_f(0 \cdot m) = 115.12\,\text{kN}$

$V_f(L_1) = -189.83\,\text{kN}$

$V_f(L_1 + 1 \cdot mm) = 161.03\,\text{kN}$

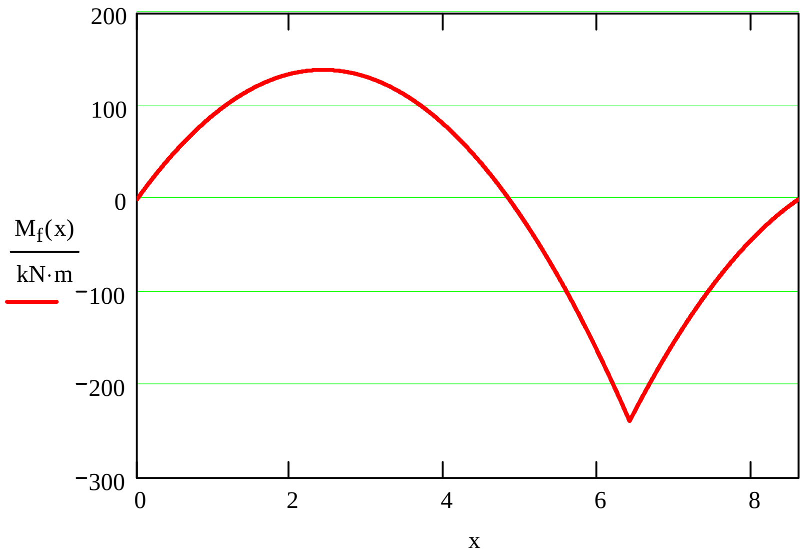
$V_f(L_1 + L_2) = 56.25\,\text{kN}$

Moment Diagram:

$$M_f(x) := \begin{cases} R_L \cdot x - 0.5 w_f \cdot x^2 & \text{if } x \le L_1 \\ R_L \cdot x - 0.5 w_f \cdot x^2 + R_R \cdot (x - L_1) & \text{otherwise} \end{cases}$$

Location of maximum moment:

$x_{max} := \frac{R_L}{w_f} \qquad x_{max} = 2.416\,\text{m}$



$M_f(x_{max}) = 139.07\,kN \cdot m$

$M_f(L_1) = -239.06\,kN \cdot m$

a) Positive moment design: $M_r := M_f(x_{max})$

$K_r := \frac{M_r}{b \cdot d^2}$ $\quad K_r = 2.01\,MPa$

$$\rho_{pos} := \frac{\left[\phi_c \cdot \alpha_1 \cdot f'_c - \left(\phi_c{}^2 \cdot \alpha_1{}^2 \cdot f'_c{}^2 - 2 \cdot K_r \cdot \phi_c \cdot \alpha_1 \cdot f'_c\right)^{\left(\frac{1}{2}\right)}\right]}{\left(f_y \cdot \phi_s\right)} \qquad \rho_{pos} = 0.00634$$

$A_{s\_pos} := \rho_{pos} \cdot b \cdot d$ $\quad A_{s\_pos} = 986.9\,mm^2$ < balanced - OK, steel yields

Try: $n_{bar} := 2$ $\quad d_b := 25 \cdot mm$ $\quad A_s := n_{bar} \cdot A_{s\_bar}(d_b)$ $\quad A_s = 1000\,mm^2$

Check beam width: $aggregate := 20 \cdot mm$

Clear spacing: $s_1 := 1.4 \cdot d_b$ $\quad s_1 = 35\,mm$ <--- Governs

$s_2 := 1.4 \cdot aggregate$ $\quad s_2 = 28\,mm$

$s_3 := 30 \cdot mm$

$s := s_1$

$b_{req} := 2 \cdot (cc + 10 \cdot mm) + n_{bar} \cdot d_b + \left(n_{bar} - 1\right) \cdot s$ $\quad b_{req} = 165\,mm$ < b = 350 mm OK

Check minimum steel area:

$$A_{smin} := 0.2 \cdot MPa \cdot \frac{\sqrt{\frac{f_c}{MPa}}}{f_y} \cdot b \cdot h \qquad A_{smin} = 479\,mm^2$$ OK

Therefore, use 2 No. 25 bars for positive reinforcement

- Negative reinforcement: $M_r := |M_f(L_1)|$ $M_r = 239.06\,kN\cdot m$

$$M_r = \phi_s \cdot A_s \cdot f_y \cdot \left(d - \frac{a}{2}\right)$$ - 2 unknowns (A_s and a)

Also: $C_c = T$ $\phi_c \cdot \alpha_1 \cdot f'_c \cdot a \cdot b = \phi_s \cdot A_s \cdot f_y$

$$a = \phi_s \cdot A_s \cdot \frac{f_y}{(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b)}$$

$$M_r = \phi_s \cdot A_s \cdot f_y \left[d - \phi_s \cdot A_s \cdot \frac{f_y}{2(\phi_c \cdot \alpha_1 \cdot f'_c \cdot b)}\right]$$

$$A_{s\_neg} := \frac{\left[d \cdot \phi_c \cdot \alpha_1 \cdot f'_c \cdot b - \left(d^2 \cdot \phi_c{}^2 \cdot \alpha_1{}^2 \cdot f'_c{}^2 \cdot b^2 - 2 \cdot M_r \cdot \phi_c \cdot \alpha_1 \cdot f'_c \cdot b\right)^{\left(\frac{1}{2}\right)}\right]}{(f_y \cdot \phi_s)}$$

$A_{s\_neg} = 1807.1\,mm^2$ **<-- Required steel area for negative reinfocement**

$\rho_{neg} := \frac{A_{s\_neg}}{b \cdot d}$ $\rho_{neg} = 0.0116$ < balanced - OK, steel yields

Try: $n_{bar} := 4$ $d_b := 25 \cdot mm$ $A_s := n_{bar} \cdot A_{s\_bar}(d_b)$ $A_s = 2000\,mm^2$

Check beam width: $aggregate := 20 \cdot mm$

Clear spacing: $s_1 := 1.4 \cdot d_b$ $s_1 = 35\,mm$ <--- Governs

$s_2 := 1.4 \cdot aggregate$ $s_2 = 28\,mm$

$s_3 := 30 \cdot mm$

$s := s_1$

$b_{req} := 2 \cdot (cc + 10 \cdot mm) + n_{bar} \cdot d_b + (n_{bar} - 1) \cdot s$ $b_{req} = 285\,mm$ < b = 350 mm OK

Check minimum steel area: $A_{smin} = 479\,mm^2$ OK

Therefore, use 4 No. 25 bars for negative reinforcement